Proofs in the Wild

What's done today?
What's close?
What's far?

Mike Dodds - Galois, Inc. - December 2024

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Context: I was an academic, then I wasn’t

2004 → 2017: *UK*

- York / Cambridge / York - PhD, postdoc, lecturer (~ associate professor)
- Logic design, automated reasoning, hardware models

2017 → now: *Portland OR*

- Galois Inc, PI / principal scientist
- Proofs for lots of different things: parsers, crypto(graphy), crypto(currency), protocols, cyber-physical systems …

Context: Galois does research for $$$

- A contract research shop / “R&D temp agency”
- 110 people, employee-owned
- Focus on security / reliability tech (PL, proof tech, static analysis)
- Clients: DARPA / DoD, some US Gov, some commercial

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Galois does proof technologies

DARPA HACMS - formally verified drone controllers

- *Built on SeL4 verified microkernel & other proof technologies*
- *Cool demo: flew an unmanned helicopter, resisted red team attack*

AWS LibCrypto - https://github.com/awslabs/aws-lc-verification

- *Proofs for crypto code from OpenSSL*
- *(Candidate for) the most heavily used bit of verified code ever*

PROVERS - current multi-$m DARPA project

- *Aim: usability for testing and proof tools*
- *Verifying cyber-physical systems as built by DoD*

Proof tech in industry is small

Low-confidence guess: <1000 proof-focused industry engineers in US

Anec-data:

- Galois is big - 60-70 technical staff
- Conferences (CAV, PLDI …) - mostly academic, 100s of attendees
- Large % engineers have PhDs, small slow-growing talent pool

Some significant teams

- AWS (biggest / most public)
- Meta / Facebook
- Hardware companies - Intel most famously
- Crypto / blockchain
- High assurance things for US Gov

What proof tech does industry actually deploy?

1. Fully-automated program analysis
2. Model checking
3. ‘White glove’ verification / interactive theorem proving

1. Fully-automated program analysis

Eliminate a particular bug category at scale, e.g:

- Memory safety issues - Infer (Facebook / Meta)
- Cloud misconfigurations - Tiros / Zelkova (AWS)

Typical tools: custom analysis tools backed by logical solvers

Trade-offs:

- (+) Scales to millions of loc, can be used by non-specialist engineers
- (-) Unsound & incomplete - false positives and false negatives. V limited properties. Tools are heuristic and specialized to particular use-cases.

2. Model checking

A small / combinatorial *[thing]* must be correct, e.g:

- Hardware - arithmetic unit on a processor
- Cryptographic primitive - AES, SHA, ECDSA

Typical tools: encode the whole system as a logical formula, solve with SMT

Trade-offs:

- (+) Fully automated, exhaustive, less need for human-written internal specifications / overrides
- (-) Scalability VERY limited, only works for small things (or things that can be reduced to small models, such as protocols)

3. ‘White glove’ verification

A mid-scale complex self-contained *[thing]* must be correct, e.g:

- Operating system kernel - SeL4, CertiKOS, BlueRock
- Cryptographic library - HACL*, AWS LibCrypto

Typical tools: interactive theorem provers, eg. Coq, Lean, F*

Trade-offs:

- (+) Extremely high level of confidence; can prove very deep properties of the system; scales to true mathematical reasoning
- (-) Required deep human effort from experts; extremely expensive per line of code; changes to the verified system are equally expensive.

Barrier to increased adoption: cost/benefit

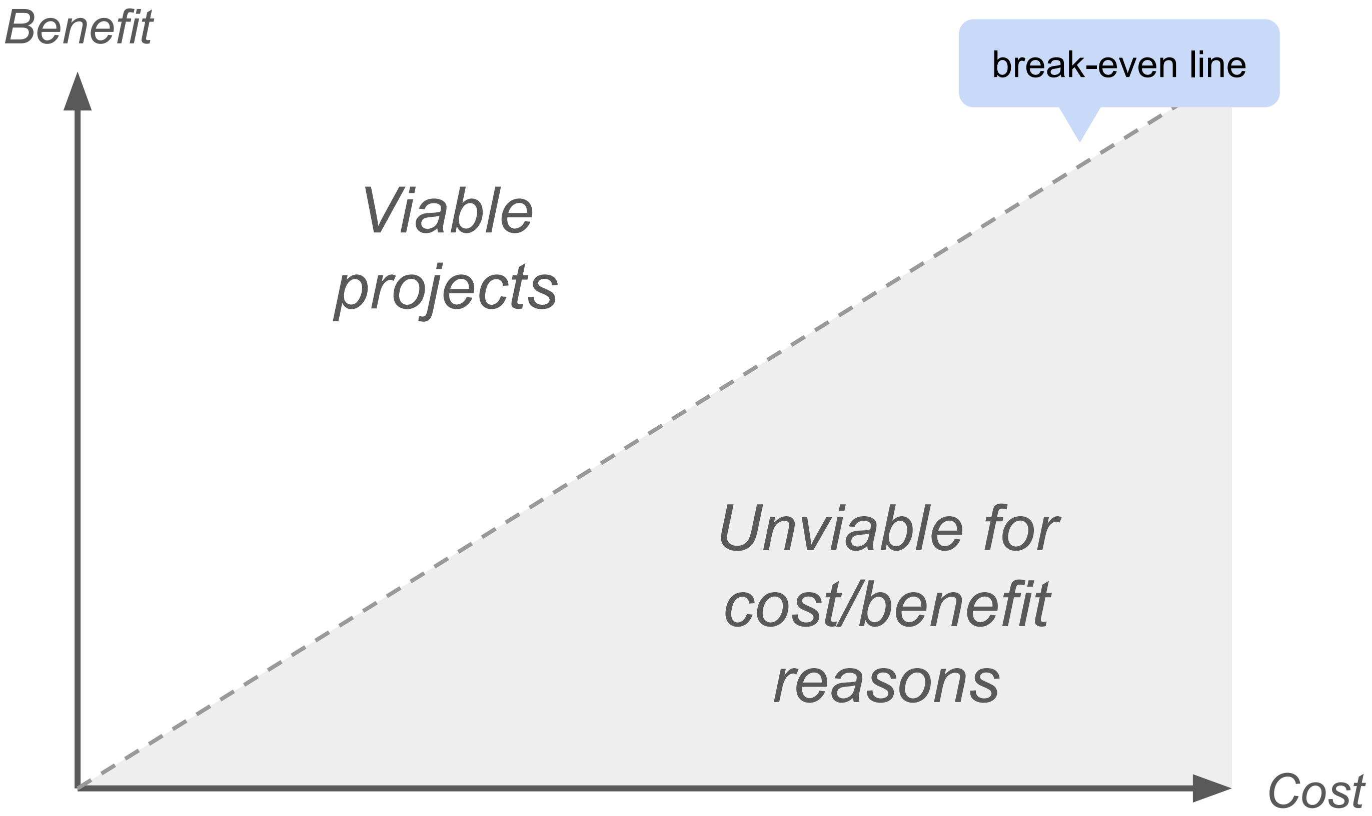
Writing proofs is very hard

- Proof scripts
- Internal function specifications / invariants
- Selection of abstractions

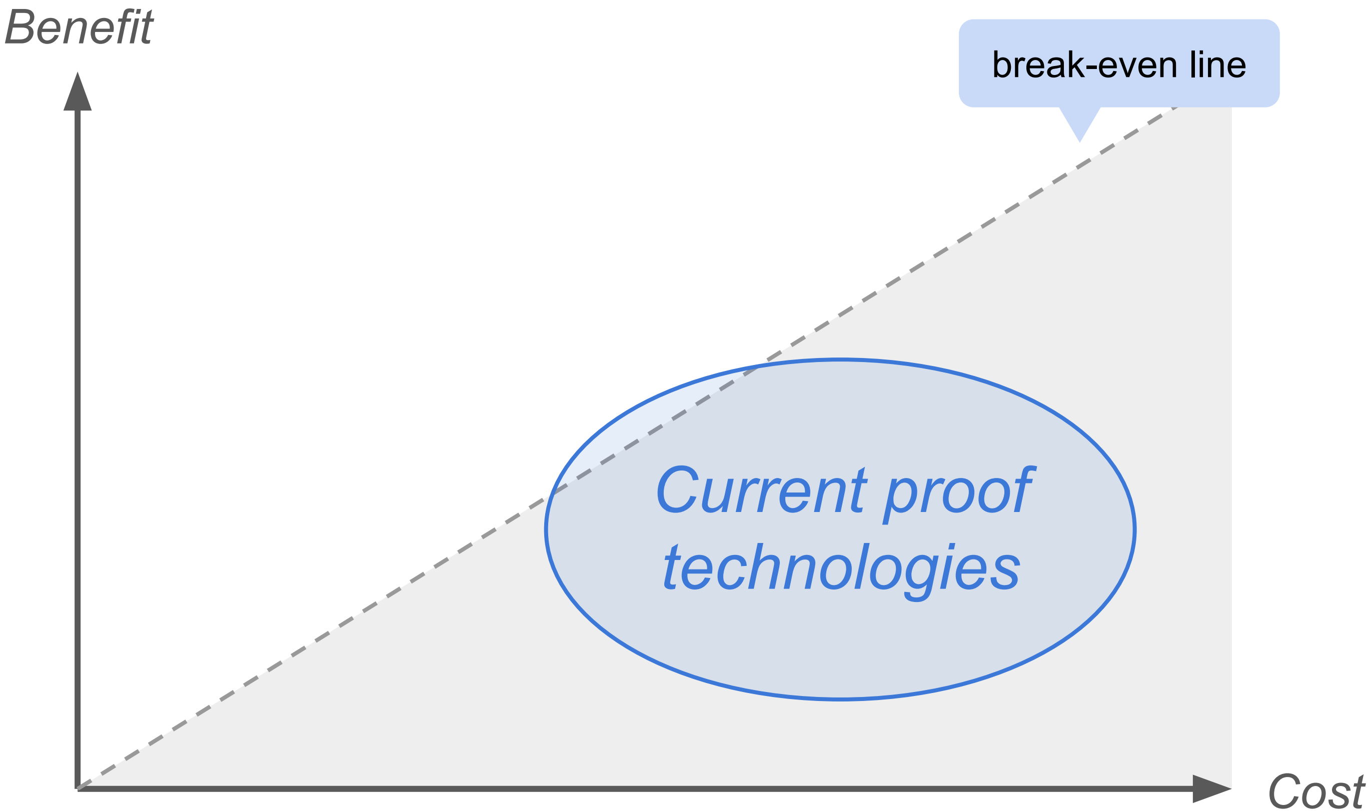
Writing specifications / world models is very hard

- Component-level specifications - pre/post conditions, reference code
- System models - language / compiler / hardware
- Environment models - threat models, user models, physics

Result: many possible projects don’t ‘pencil out’



Result: many possible projects don’t ‘pencil out’



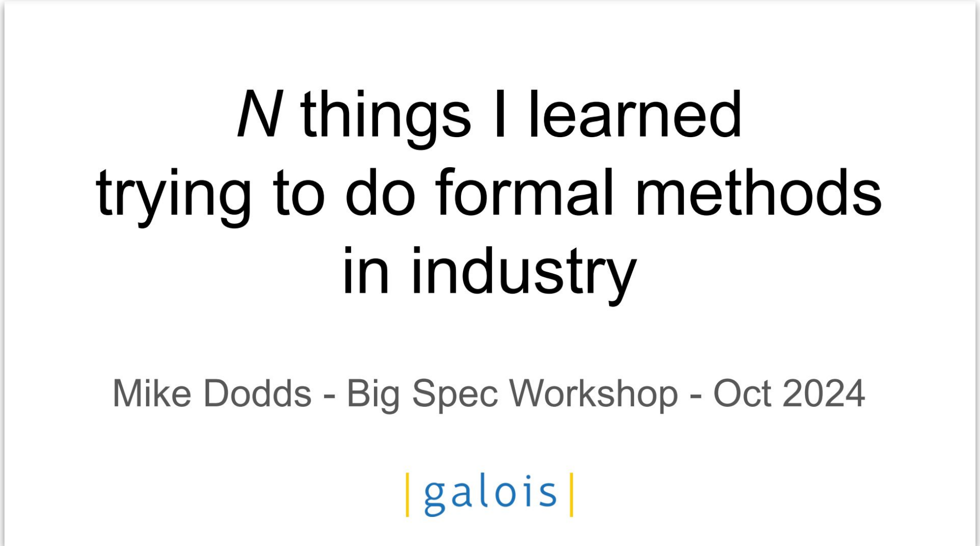
Success stories have solved this by careful scoping

Eg:

- Making properties very restricted
- Targeting very small systems
- Spending huge amounts of labor

Worth it for some very critical problems!

More on the cost/benefit landscape for proof tech:



N things I learned
trying to do formal methods
in industry

Mike Dodds - Big Spec Workshop - Oct 2024

galois

https://mikedodds.github.io/files/talks/2024-10-09-n-things-I-learned.pdf

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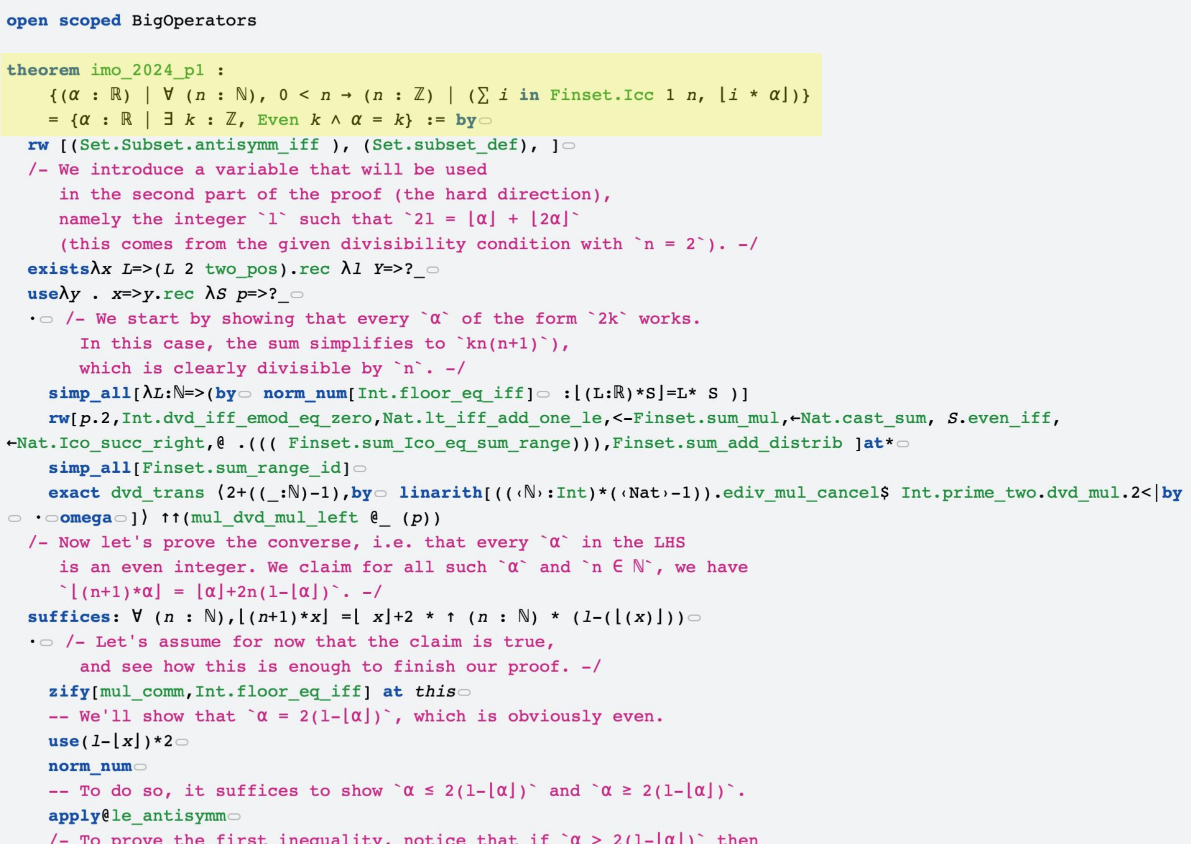
AI-driven proof

Writing proof scripts is arduous

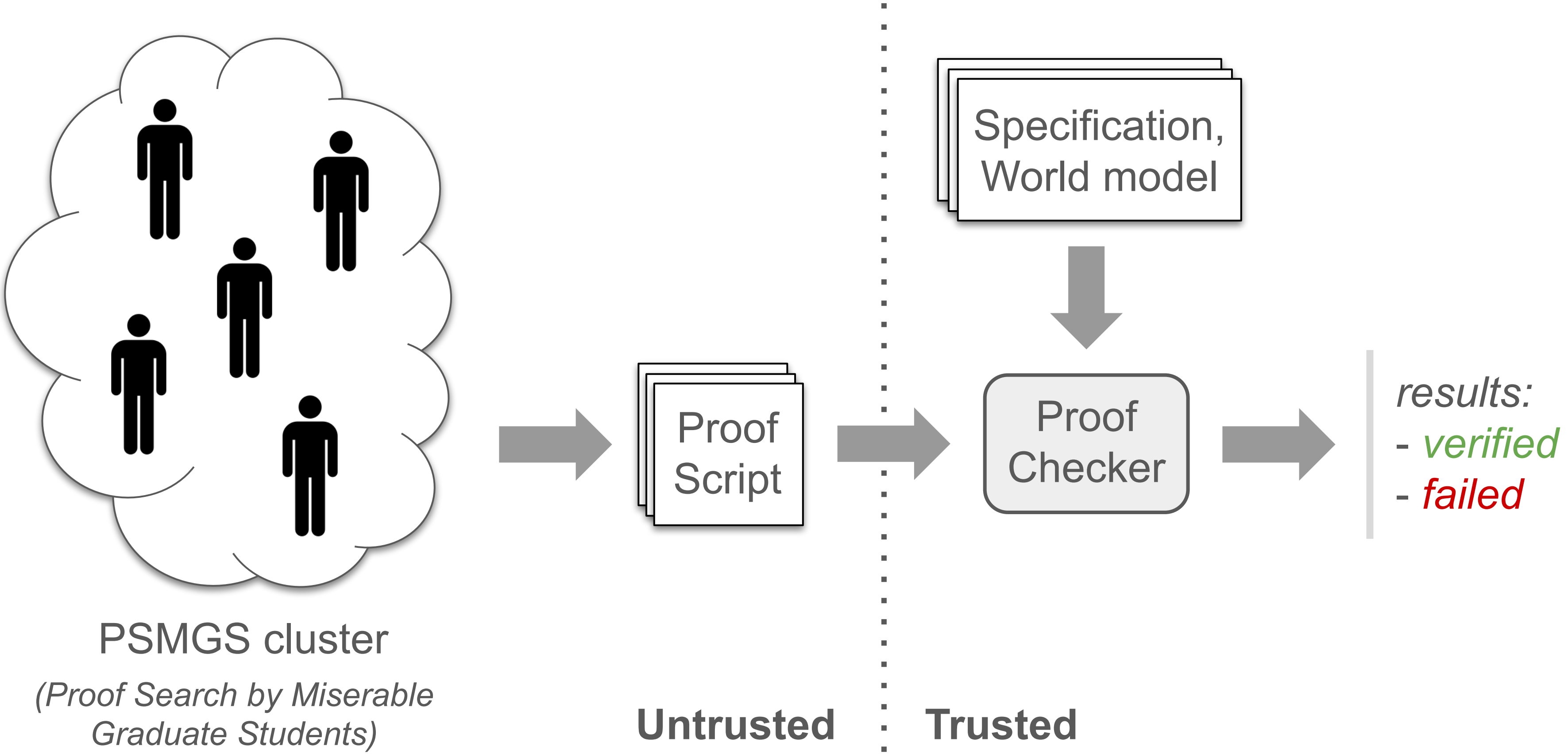


```
open scoped BigOperators

theorem imo_2024_p1 :
    {(α : ℝ) | ∀ (n : ℕ), 0 < n → (n : ℤ) ∣ (∑ i in Finset.Icc 1 n, ⌊i * α⌋)}
    = {α : ℝ | ∃ k : ℤ, Even k ∧ α = k} := by⬭
  rw [(Set.Subset.antisymm_iff ), (Set.subset_def), ]⬭
  /- We introduce a variable that will be used
    in the second part of the proof (the hard direction),
    namely the integer `l` such that `2l = ⌊α⌋ + ⌊2α⌋`
    (this comes from the given divisibility condition with `n = 2`). -/
  existsλx L=>(L 2 two_pos).rec λl Y=>?_⬭
  useλy . x=>y.rec λS p=>?_⬭
  ·⬭ /- We start by showing that every `α` of the form `2k` works.
      In this case, the sum simplifies to `kn(n+1)`),
      which is clearly divisible by `n`. -/
    simp_all[λL:ℕ=>(by⬭ norm_num[Int.floor_eq_iff]⬭ :⌊(L:ℝ)*S⌋=L* S )]
    rw[p.2,Int.dvd_iff_emod_eq_zero,Nat.lt_iff_add_one_le,<-Finset.sum_mul,←Nat.cast_sum, S.even_iff,
←Nat.Ico_succ_right,@ .((( Finset.sum_Ico_eq_sum_range))),Finset.sum_add_distrib ]at*⬭
    simp_all[Finset.sum_range_id]⬭
    exact dvd_trans ⟨2+((_:ℕ)-1),by⬭ linarith[((‹ℕ›:Int)*(‹Nat›-1)).ediv_mul_cancel$ Int.prime_two.dvd_mul.2<|by
⬭ ·⬭omega⬭]⟩ ↑↑(mul_dvd_mul_left @_ (p))
  /- Now let's prove the converse, i.e. that every `α` in the LHS
    is an even integer. We claim for all such `α` and `n ∈ ℕ`, we have
    `⌊(n+1)*α⌋ = ⌊α⌋+2n(l-⌊α⌋)`. -/
  suffices: ∀ (n : ℕ),⌊(n+1)*x⌋ =⌊ x⌋+2 * ↑ (n : ℕ) * (l-(⌊(x)⌋))⬭
  ·⬭ /- Let's assume for now that the claim is true,
      and see how this is enough to finish our proof. -/
    zify[mul_comm,Int.floor_eq_iff] at this⬭
    -- We'll show that `α = 2(l-⌊α⌋)`, which is obviously even.
    use(l-⌊x⌋)*2⬭
    norm_num⬭
    -- To do so, it suffices to show `α ≤ 2(l-⌊α⌋)` and `α ≥ 2(l-⌊α⌋)`.
    apply@le_antisymm⬭
    /- To prove the first inequality, notice that if `α > 2(l-⌊α⌋)` then
```

Google DeepMind, IMO 2024 Problem 1. https://storage.googleapis.com/deepmind-media/DeepMind.com/Blog/imo-2024-solutions/P1/index.html

Classic interactive theorem proving architecture



This is just a search process!

(untrusted)

Guess

- *Expensive*
- *Stochastic*
- *Hard to audit*

(trusted)

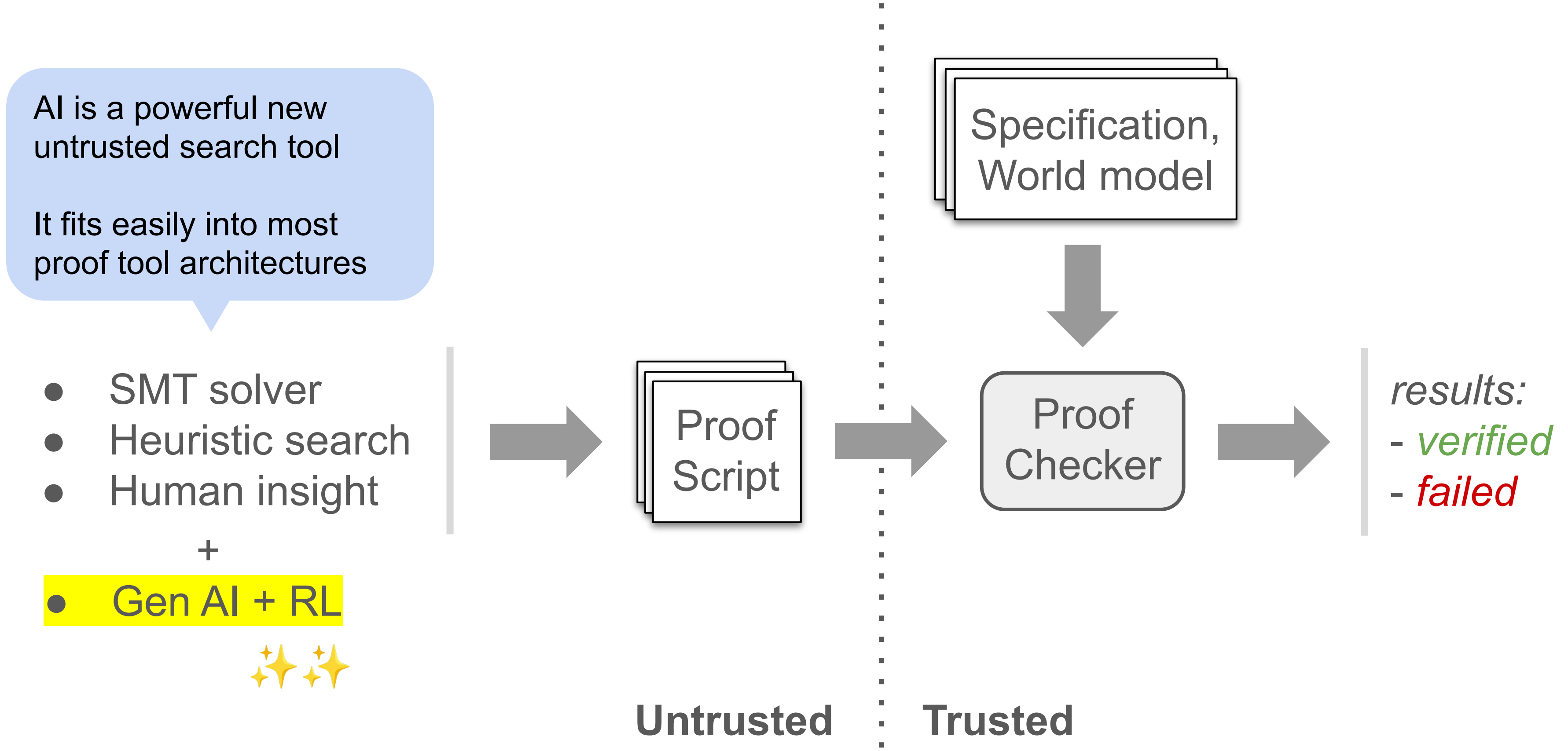
Check

- *Cheap*
- *Deterministic*
- *Easy to audit*

Many proof tech problems are just *search*

Guess		***Check***
Write a proof script	→	Check proof establishes the theorem
Add types to a program	→	Typecheck the program
Write program invariants	→	Check the program verification
Synthesize a program that matches a specification	→	Check the program matches the specification
[Heuristic generator]	→	**[Trusted checker]**

Almost all proof tools are ~structured this way



AI is a powerful new untrusted search tool

It fits easily into most proof tool architectures

- SMT solver
- Heuristic search
- Human insight

+

- Gen AI + RL

✨✨

Proof Script

Specification, World model

Proof Checker

results:

- *verified*
- *failed*

Untrusted

Trusted

Optimism: AI proofs get really cheap

Early indicators:

- AlphaProof IMO - automated proof search for v hard problems
- Towards Neural Synthesis for SMT-Assisted Proof-Oriented Programming, Microsoft Research https://arxiv.org/abs/2405.01787

Optimism: AI proofs improve rapidly

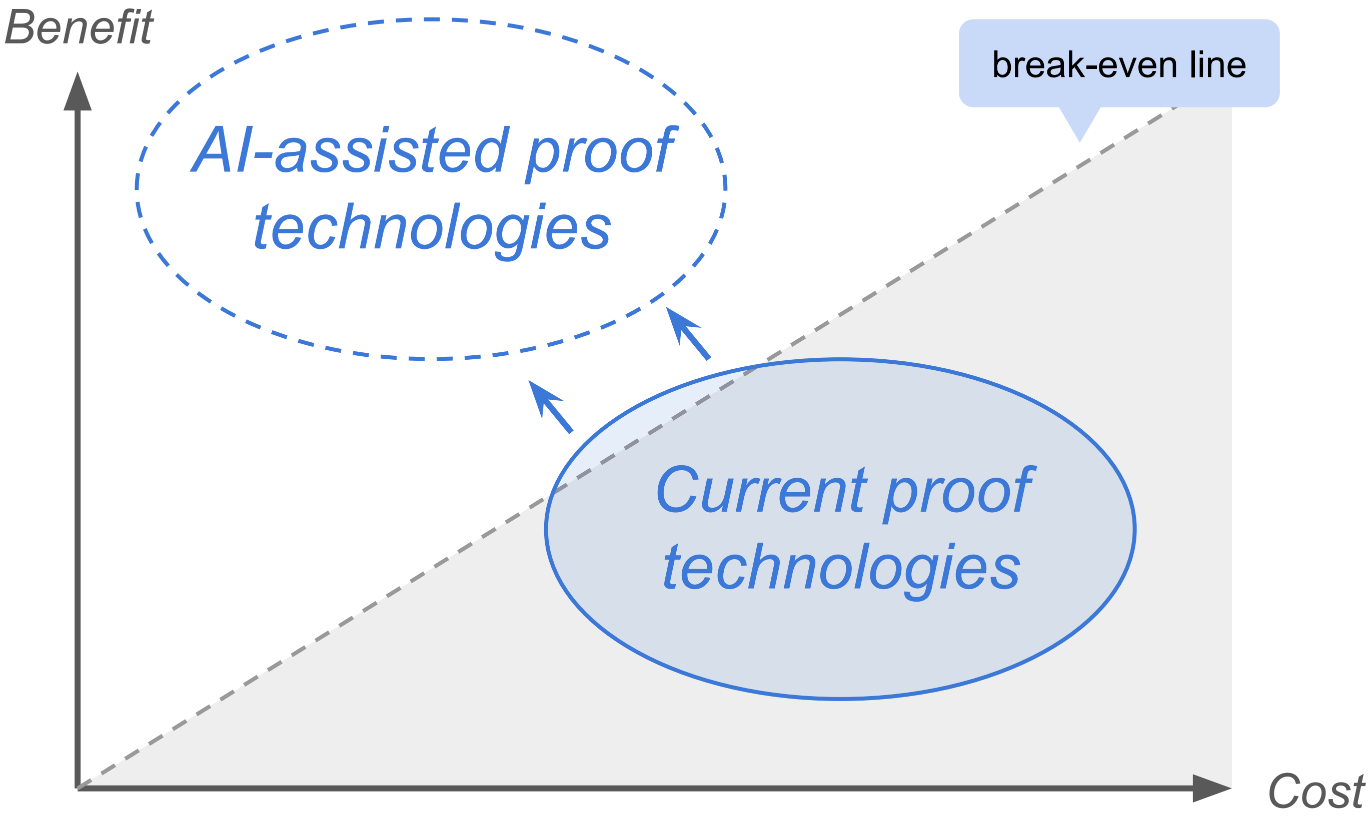
Synthetic data / RL

- Proof tools are a totally reliable oracle of correct / incorrect proofs
- Oracle + LLM + RL - seems promising for synthetic proof data generation

Current proof datasets are small

- Making proof easier should result in more proof data written by users
- Virtuous cycle - increased datasets result in improved capabilities

Optimism: many more proof technologies get useful



Optimism: impossible things become possible

Eg:

- Auto-coders that ‘certify their work’, generating proofs alongside diffs
- Transpile 10s of millions of lines of C with memory safety guarantees
- Insert proved-correct security boundaries into legacy systems
- Retrofit a Linux-scale operating system with proofs

These are *in a sense* currently possible, just much too expensive

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What’s done today?

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What’s far?

Specifications and world models

Current specification technologies

Mostly discrete, bounded, logical

- Logical formulas (+ various fancy extensions)
- State machines
- Domain specific languages

Eg. Cerberus: https://www.cl.cam.ac.uk/~pes20/cerberus/

- A highly accurate model of the C programming language
- Captured in a DSL called Lem which encodes logical states and updates
- Several person-years of iteration: building / testing / discussing

Formal specifications, ideally:

Mathematically clean

Stable over time

Agreed by the users of the system

Easy to reason about

Big successes ALL fit this ideal model

- Cryptographic algorithms
- Operating systems / hypervisors
- Compilers / programming languages
- Cloud services
- Hardware

The reality:

- These systems are *unusually easy to specify*
- Even slightly harder-to-specify things are very hard to deal with

Most real-world specifications are not…

Mathematically clean

Stable over time

Agreed by all users of the system

Easy to reason about

Real-world specifications are very non-formalisable

- Prose standards / RFCs / papers
- Powerpoint decks *(v common)*
- The code itself
- Reference implementations
- Inline code comments
- Test cases
- User stories
- Requirements documents
- Regulatory rules
- Scribbled notes on coffee-shop napkins
- …

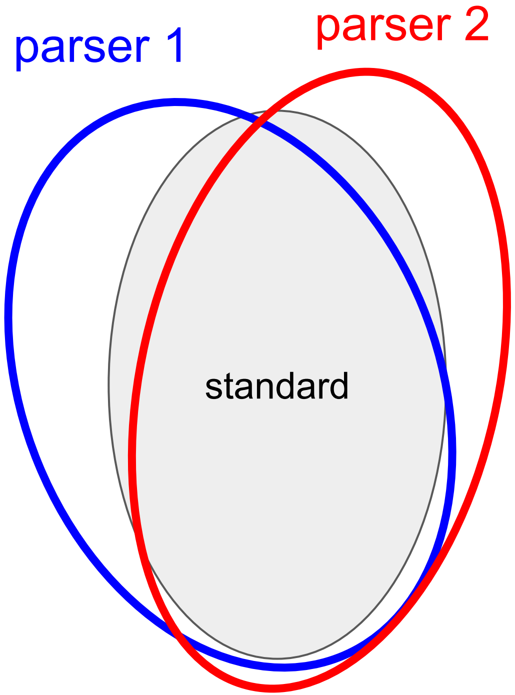
Anecdote: PDF, a spec that does not exist

We formalized PDF in our format definition language Daedalus (https://github.com/GaloisInc/daedalus)

- Testing on millions of cases
- Worked closely with the PDF association

But…

- Non-descriptive: different from real parsers
- Non-normative: doesn't characterize bugs
- Unclear how to get to a more rigorous & accepted specification



We’ve only explored the easiest classes of spec

Cryptographic algorithm

Operating system

Document format

CPS system, eg nuclear reactor

Web browser

AI-driven chemical synthesis tool

Generic conversational AI

Increasingly:

- *Complex*
- *Ambiguous*
- *Hard to reason about*
- *Contended by users*
- *‘Open world’*

We’ve only ex classes of spec

We only really have examples of these two levels in industry use

Cryptographic algorithm

Operating system

Document format

CPS system, eg nuclear reactor

Web browser

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Generic conversational AI

Increasingly:

- *Complex*
- *Ambiguous*
- *Hard to reason about*
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Eg. 1: operating system verification

Specification: “Data should not flow from high to low security domains”

Approach (similar to SeL4):

- Tag data with security levels
- Model operating system operations via logic
- Prove that each operation preserves security invariants

Challenges:

- Specification: what user-side behaviors are possible?
- World modelling: are hardware / physics behaviors in scope?

… vs Eg. 2: AI-driven chemical synthesis tool

Specification: “Do not generate chemicals that harm humans”

Approach:

- Write a model of ‘harmful chemicals’
- Prove some guard system correctly rejects all such chemicals

Challenges:

- Need a granular probabilistic model of chemistry and human biology
- “Harm” is a socio-technical term - need to capture social convention / law
- “Harm” may include combined chemicals, so we need a compositional theory how chemicals could be used

Optimism: can probabilistic programming help?

Maybe? My sense is the tech is very early

Hard problems:

- How do we reason about probabilities at scale?
- How do we validate models vs the real world, esp. over time?
- Is probabilistic reasoning valid in the presence of adversarial actors?

Optimism: can AI help?

Plausible ideas:

- AI + human teaming on specification writing
- AI-driven science to develop accurate models of the world

A lot of work is needed on ‘spec tech’

We have a 50+ years of tools for easy-to-specify things

~Zero tools for hard-to-specify things

For GSAI:

- Big divide between plausible cases and ‘science fiction’
- Urgent need to experiment / grow the bench
- Unclear if / what progress is being made

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Summary

https://mikedodds.github.io

What’s done today:

- A small number of successful proof tech deployments
- Strong evidence of usefulness in some domains
- A deep bench of tools and ideas, though many are too expensive
- Key barrier is cost/benefit - proofs are hard and specs are hard

What’s close: proofs

- AI is great for proof search!
- Current tool architectures can integrate AI with very little modification
- *Optimism:* proofs get cheap, proof tech gets much more useful

What's far: specifications / world models

- Current proof tech focuses on a tiny range of easy-to-specify things
- We have ~zero examples of success in more difficult-to-specify domains
- Spec tech needs rapid development if we expect to apply it soon (per GSAI)

Thanks!

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N things I learned trying to do formal methods in industry:

https://mikedodds.github.io/files/talks/2024-10-09-n-things-I-learned.pdf